EXERCISE – V JEE PROBLEMS

1. (a) Consider an infinite geometric series with first term a and common ratio r. If the sum is 4 and the second term is 3/4, then **[JEE 2000, (scr.), 1 + 1]**

(A) $a = \frac{4}{7}, r = \frac{3}{7}$ (B) $a = 2, r = \frac{3}{8}$

(C) $a = \frac{3}{2}, r = \frac{1}{2}$ (D) $a = 3, r = \frac{1}{4}$

(b) If a, b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisifes the relation :

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$

(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

(c) The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[JEE 2000, (Mains), 4]**

2. Given that α, γ are roots of the equation, $Ax^2-4x+1=0$ and β, δ the roots of the equation, $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P. **[REE 2000, 5]**

3. The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P. ? **[REE 2001, 3]**

4. Solve the following equations for x and y

$\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots\ldots\ldots.$

$\ldots\ldots\ldots\ldots = y \dfrac{5+9+13+\ldots\ldots\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots\ldots\ldots+(2y-1)} = 4\log_4 x$

[REE 2001, 5]

5. (a) Le α, β be the roots of $x^2 - x + p = 0$ and γ, δ the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6. – 32

(b) If the sum of the first 2n terms of the A.P. 2, 5, 8,… ……….is equal to the sum of the first n terms of the A.P. 57, 59, 61,……., then n equals

(A) 10 (B) 12 (C) 11 (D) 13

(c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are **[JEE 2001, (Scr.) 1 + 1 + 1]**

(A) NOT in A.P./G.P./ H.P. (B) in A.P.

(C) in G.P. (D) in H.P.

(d) Let $a_1, a_2, \ldots\ldots$ be positive real numbers in G.P. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, \ldots\ldots\ldots, a_n$. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots\ldots, G_n$ in terms of $A_1, A_2, \ldots\ldots\ldots, A_n$, $H_1, H_2, \ldots\ldots, H_n$. **[JEE 2001 (Mains), 5]**

6. (a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if a < b < c and a + b + c = 3/2, then the value of a is **[JEE 2002 (Scr.), 3]**

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

(b) Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P. show that $\dfrac{G_1G_2}{H_1H_2} = \dfrac{A_1+A_2}{H_1+H_2} = \dfrac{(2a+b)(a+2b)}{9ab}$.

[JEE 2002 (Mains), 5]

7. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either a = b = c or a, b, $-\frac{c}{2}$ form a G.P.

[JEE 2003 (Mains), 4]

8. The first term of an infinite geometric progression is x and its sum is 5. Then **[JEE 2004 (Scr.)]**

(A) $0 \le x \le 10$ (B) $0 < x < 10$

(C) $-10 < x < 0$ (D) $x > 10$

9. If a, b, c are positive real numbrs. then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4b^4c^4$. **[JEE 2004, 4]**

10. (a) In the quadratic equation $ax^2 + bx + c = 0$, If $\Delta = b^2 - 4ac$ and $\alpha+\beta, \alpha^2+\beta^2, \alpha^3+\beta^3$, are in G.P. where α, β are the roots of $ax^2 + bx + c = 0$, then

[JEE 2005 (Scr.)]

(A) $\Delta \ne 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

(b) If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)$ $(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \le k \le n$. Find n. **[JEE 2005 (Mains), 2]**

11. If $A_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ...(-1)^{n-1}\left(\frac{3}{4}\right)^n$

and $B_n = 1 - A_n$, then find the minimum natural number n_0 such that $B_n > A_n \ \forall\, n > n_0$. **[JEE 2006, 6]**

12. Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1, 2,.....

(a) The sum $V_1 + V_2 + ... + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n+1)$ (B) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(C) $\frac{1}{2}n\,(2n^2 - n+1)$ (D) $\frac{1}{3}(2n^3 - 2n+3)$

(b) T_r is always

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

(c) Which one of the following is a correct statement ?

(A) Q_1, Q_2, Q_3,.... are in A.P., with common difference 5

(B) Q_1, Q_2, Q_3,.... are in A.P., with common difference 6

(C) Q_1, Q_2, Q_3,.... are in A.P., with common difference 11

(D) $Q_1 = Q_2 = Q_3 =$.... **[JEE 2007, 4 + 4 + 4]**

13. Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \ge 2$, Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively

(a) Which one of the following statements is correct ?

(A) $G_1 > G_2 > G_3 > ....$

(B) $G_1 < G_2 < G_3 < ....$

(C) $G_1 = G_2 = G_3 = ...$

(D) $G_1 < G_3 < G_5 < ....$ and $G_2 > G_4 > G_6 > ....$

(b) Which one of the following statement is correct ?

(A) $A_1 > A_2 > A_3 > ....$

(B) $A_1 < A_2 < A_3 < ....$

(C) $A_1 > A_3 > A_5 > .....$and $A_2 < A_4 < A_6 < ....$

(D) $A_1 < A_3 < A_5 < .....$and $A_2 > A_4 > A_6 > ....$

(c) Which one of the following statement is correct ?

(A) $H_1 > H_2 > H_3 > ....$

(B) $H_1 < H_2 < H_3 < ....$

(C) $H_1 > H_3 > H_5 > .....$and $H_2 < H_4 < H_6 < ....$

(D) $H_1 < H_3 < H_5 < .....$and $H_2 > H_4 > H_6 > ....$

[JEE 2007, 4 + 4 + 4]

14. (a) A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then **[JEE 2008, 4]**

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (B) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (D) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

(b) Supose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$. **[JEE 2008, 3]**

Statement–1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

Statement–2 : The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement (1) is true and statement (2) is true and statement (2) is correct explanation for (1)

(B) Statement (1) is true and statement (2) is true and statement (2) is NOT correct explanation for (1)

(C) Statement (1) is true but (2) is false

(D) Statement (1) is false but (2) is true

15. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is **[JEE 2009, 3]**

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

16. Let S_k, K = 1, 2, ..., 100 denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is 1/k. Then the value of

$\frac{100^2}{100!} + \sum_{k=1}^{100} \left|(k^2 - 3k + 1)S_k\right|$ is **[JEE 2010]**

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17. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$

If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to **[JEE 2010]**

18. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is **[JEE 2011]**

19. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$. let $m=5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011]**

20. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[JEE 2012]**

(A) 22 (B) 23 (C) 24 (D) 25

Sol.